Numerical solutions of range-dependent benchmark problems in ocean acoustics

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Accurate numerical solutions are presented for benchmark problems associated with propagation in a wedge with different bottom boundary conditions as well as in a plane-parallel waveguide with changing profile with range. The benchmark problems are solved with three different general-purpose numerical codes: (1) a coupled-mode code providing a full-spectrum two-way (backscatter included) solution of the wave equation, (2) a finite-difference PE code providing a wide-angle solution of the one-way parabolic approximation of the wave equation, and (3) a split-step PE code providing a more narrow-angle solution of the one-way parabolic approximation of the wave equation. Only the two-way coupled-mode results can be considered accurate solutions of the benchmark problems.

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INTRODUCTION

The field of numerical modeling of sound propagation has been in continuous expansion over the past 20 years[1] trailing closely the stunning advances in computer technology. Up till around 1970, the only practical technique for solving propagation problems in underwater acoustics was based on ray theory, which is a computationally efficient but approximate (infinite-frequency) solution of the wave equation. The ray techniques are still in use for solving high-frequency deep-water problems in ocean acoustics.

In the early 1970s, powerful digital computers became available in most research establishments, stimulating the development of more accurate frequency-dependent solutions of the wave equation. These wave-theory solutions (all numerically based) encompass normal mode, fast field, and parabolic equation techniques.[1] Today general-purpose numerical codes based on the above solution techniques are widely used for computing the acoustic field in complex ocean environments.

Since closed-form analytic solutions are not available for checking the numerical results even for the most simple range-dependent environments, we are confronted with the fundamental problem of *how to ascertain that a numerical solution generated by a complex computer program after hours of calculation on a multimegaflop machine is an accurate solution of the posed mathematical problem*. There seems to be no simple solution to this problem so there is always some question about the accuracy of published numerical results, even when the author has exercised his code with the utmost care.

Some useful steps in a general validation procedure for range-dependent acoustic models are: comparison with analytic reference solutions for range-independent environments; check of energy conservation and reciprocity in the solution; intermodel comparison; and comparison with numerical benchmark solutions for range-dependent environments.

The first two steps are easily carried out by the model developer or the model user. The third step requires access to alternative models applicable to a given propagation problem, but it certainly is a valid check of the implementation of a particular numerical solution. The last point is also based on intermodel comparisons, but here the test problems are carefully selected and a variety of numerical solutions compared in order to arrive at "accepted" reference solutions, which in turn are published with stated accuracy bounds. Collectively, these steps serve the general purpose of validating a given model and ultimately of improving confidence in published numerical results.

A successful attempt to establish relevant test problem solutions for parabolic equation codes was done in connection with the PE Workshop held at NORDA in 1981. Several low-frequency propagation problems were solved with a variety of numerical codes, and reference solutions were identified.[2,3] These PE test problems have been extensively used in the community over the past 8 years.

Recently, the issue of establishing reference solutions for range-dependent ocean acoustic problems was addressed within the Acoustical Society of America (ASA). Special sessions at two consecutive ASA meetings were dedicated to this problem, and relevant benchmark problems were identified and solved.[4] What we present in this paper are our solutions of the ASA benchmark problems, generated with three different general-purpose numerical codes.

Since wave-theory techniques are computationally slow at high frequencies, the ASA benchmark problems concentrate on low-frequency propagation. There is, however, an equal need for high-frequency reference solutions, but this topic was deferred to the future. Moreover, only simple fluid bottoms are considered, leaving also the solution of the elastic range-dependent problem to the future.

The accuracy requirement (a fraction of a decibel) is of overwhelming concern in the generation of test problem solutions, while the calculation time is of little relevance. This is quite contrary to the practical use of acoustic models in

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data analysis, where modeling accuracy of a few decibels is often sufficient but where fast computations are essential. However, by pushing the numerical model to the extreme in terms of accuracy, bugs in the implementation might show up, which otherwise would not have been detected. Actually, we found errors in two of our codes during the generation of accurate benchmark solutions. The errors were small and unimportant for practical modeling work, but they could conceivably have distorted the solution significantly under different propagation situations. As a result of having further debugged our codes, we have now increased confidence in their performance.

The organization of this paper is as follows: In Sec. I, we describe the selected range-dependent benchmark problems, three associated with a wedge-shaped waveguide, and one with a plane-parallel waveguide. Section II deals with the three numerical models used to solve the benchmark problems, including theoretical foundation, numerical implementation, and procedures for obtaining stable numerical results. The benchmark solutions are presented in Secs. III–VI, and the paper ends with a summary and conclusions.

I. THE BENCHMARK PROBLEMS

Range-dependent ocean scenarios generally include changes in both water depth and sound-speed profile with range. The ASA benchmark problems[4] were selected so as to study these two range-dependent features separately. First, we consider propagation upslope in a wedge-shaped homogeneous waveguide, and, second, propagation in a plane-parallel waveguide with range-varying sound-speed structure.

A. Wedge-shaped waveguide

The geometry for the wedge problem is given in Table I and graphically illustrated in Fig. 1. The environment consists of a homogeneous water column ($c = 1500$ m/s, $\rho = 1$ g/cm^3) limited above by a pressure-release flat sea surface

TABLE I. Benchmark problems associated with a wedge-shaped waveguide.

1. BENCHMARK WEDGE PROBEMS

Accurate solutions are invited for upslope acoustic propagation in a wedge with the geometry described below. The parameters of the problem are listed below, including three choices of bottom boundary condition (pressure release, case I and two penetrable bottoms, cases II and III). The wedge geometry is shown in Fig. 1.

Parameters common to all three cases.

wedge angle $\theta_0 = 2.86°$
frequency $f = 25$ Hz
isovelocity sound speed in water column $c_1 = 1500$ m/s
source depth = 100 m
source range from the wedge apex = 4 km
water depth at source position = 200 m
pressure-release surface

Case I: pressure-release bottom.

This problem should be done for a line source parallel to the apex i.e., 2-D geometry.

Case II: penetrable bottom with zero loss.

sound speed in the bottom $c_2 = 1700$ m/s
density ratio $\rho_2/\rho_1 = 1.5$
bottom attenuation = 0 dB/λ

This problem should be done for a point source in cylindrical geometry.

Case III: penetrable lossy bottom.

As in case II except with bottom loss = 0.5 dB/λ

OUTPUT

Plots should be presented on overhead transparencies of propagation loss versus range measured from the source to the apex. The scaling should be as shown. It is important to conform to this format for the purpose of comparison. The dB scale of propagation loss should cover exactly 50 dB. The start and end points of this 50-dB scale should be chosen to ensure that the results are entirely contained in the plot. Propagation loss is defined for the present purpose as

$$\mathrm{PL} = -10 \log_{10} \frac{(\text{Intensity at a field point})}{(\text{Intensity at one meter away from source})}$$

Receiver depths

Case I: 30 m
Cases II and III: 30 and 150 m

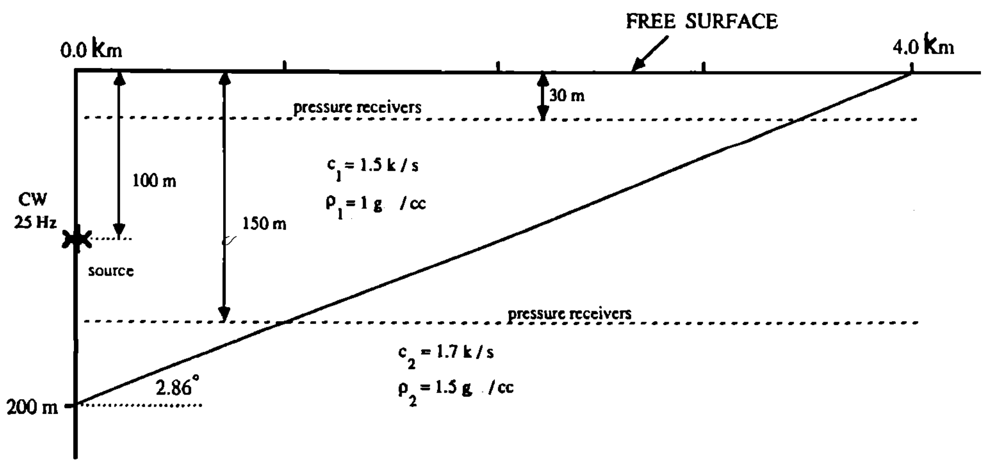

FIG. 1. Wedge geometry for test problems 1, 2, and 3.

and below by a sloping sea floor. The water depth at the source position is 200 m decreasing to zero at a distance of 4 km from the source with a slope of approximately 2.86°. Accurate field solutions are sought for a 25-Hz source placed at middepth (100 m) and for two receivers at 30- and 150-m depth, respectively. As seen from Fig. 1, the selected receiver depths provide samples of the acoustic field in the water column as well as in the bottom. The points in range where the two receivers cross the water/bottom interface is 3.4 km for the shallow receiver and 1.0 km for the deep receiver.

Three different bottom boundary conditions are considered.

Case 1: Perfectly reflecting pressure-release bottom. This is an idealized wedge problem for which an independent reference solution can be formulated. The problem should be solved in plane geometry (line source) with a radiation condition applied at the left boundary.

Case 2: Penetrable lossless bottom. This is a slightly idealized ocean acoustic problem where attenuation in the bottom has been neglected. The bottom is a homogeneous fluid half-space with a compressional speed of 1700 m/s and a density of 1.5 g/cm^3. As is customary in ocean acoustics, this problem should be solved for a point source in cylindrical geometry.

Case 3: Penetrable lossy bottom. This is a more realistic ocean acoustic problem where a wave attenuation of 0.5 dB/λ in the bottom has been included. Otherwise parameters are the same as in case 2.

The reason for considering three different benchmark problems for the wedge geometry shall be briefly explained. Case 3 is the most realistic ocean acoustic problem (lossy penetrable bottom). However, some numerical codes only handle lossless media, and case 2 (lossless penetrable bottom) was therefore included as an alternative test problem. Case 1 (perfectly reflecting bottom) was chosen simply because an analytic reference solution can be formulated for the pressure-release wedge. Moreover, since sound propagating towards the wedge apex will be completely backscattered due to the reflecting boundaries, this test problem is an ideal benchmark for a full two-way solution of the wave equation.

B. Plane-parallel waveguide

This test problem is summarized in Table II and the sound-speed profile variation with range is graphically illustrated in Fig. 2. We are considering an idealized waveguide with a pressure-release upper boundary and a rigid lower boundary. The sound speed in the waveguide varies with both depth and range according to the formula given in Table II. Solutions were solicited[4] for a low-frequency shallow-water situation (case 4) as well as for a high-frequency deep-water situation (case 5). However, since case 5 turned out to be computationally prohibitive for the numerical models on hand, we shall here consider just the shallow-water low-frequency problem, for which the water depth is 500 m and the frequency 25 Hz. Accurate field solutions are sought for both source and receiver at middepth (250 m) in cylindrical geometry.

TABLE II. Benchmark problems associated with a plane-parallel waveguide.

2. BENCHMARK RANGE-DEPENDENT PLANE-PARALLEL WAVEGUIDE

Solutions are invited for the sound field (velocity potential) in a plane-parallel waveguide with one pressure release boundary and one rigid boundary. The sound-speed profile shows a range dependence given by

$$\frac{c(r,z)}{c_0} = \left\{1 + \left(\frac{\pi l_1}{L}\right)^2 e^{-2\pi r/L} + \left(\frac{2\pi l_2}{L}\right)^2 e^{-4\pi r/L} - \frac{2\pi l_1}{L}\left[1 - \left(\frac{2\pi l_2}{L}\right)e^{-2\pi r/L}\right]\cos\left(\frac{\pi z}{L}\right)e^{-\pi r/L} - \left(\frac{4\pi l_2}{L}\right)\cos\left(\frac{2\pi z}{L}\right)e^{-2\pi r/L}\right\}^{-1/2}, \quad (1)$$

where $c_0 = 1500$ m/s is a reference sound speed, z is depth below the pressure-release surface, L is the channel depth, and r is range from the source. l_1 and l_2 are parameters with the values:

$l_1/L = 0.032 \quad l_2/L = 0.016$

Consider two cases:

Case I: (low-frequency, shallow water)

frequency: $f = 25$ Hz
$L = 500$ m

Case II: (high-frequency, deep water)

frequency: $f = 100$ Hz
$L = 3$ km

OUTPUT

Case I: As before except that the abscissa (i.e., the range coordinate) should be 5 cm/km, giving a total range coverage of 4 km.

Case II: As before except that the abscissa (i.e., the range coordinate) should be 5 cm/5 km, giving a total range coverage of 20 km.

The field should be computed for both source and receiver at depth $z = L/2$.

Figure 2 shows the change in sound-speed profile with range for case 4. Even though we consider propagation over a total range of 4 km, we note that the significant profile changes occur within the first 400 m. Thus the initial profile with a maximum speed of 1877 m/s at the surface and a minimum speed of 1346 m/s near middepth, rapidly changes to an almost isospeed profile (1500 m/s). We shall present solutions for two different source conditions: *case 4a*, a beam-limited source of width $\pm 43°$ obtained by summing the first 10 source modes; and *case 4b*, an omnidirectional source obtained by summing all 17 source modes.

The above test problem clearly does not represent a realistic ocean acoustic environment. Both the rigid bottom boundary condition and the extreme sound speed variations with depth and range within the waveguide are physically unrealistic. However, this problem was chosen because an independent reference solution can be formulated for this particular environment.

II. THE NUMERICAL MODELS

Numerical implementations of wave-theory solutions for range-dependent acoustic problems can be classified as: normal-mode techniques (adiabatic or coupled modes);

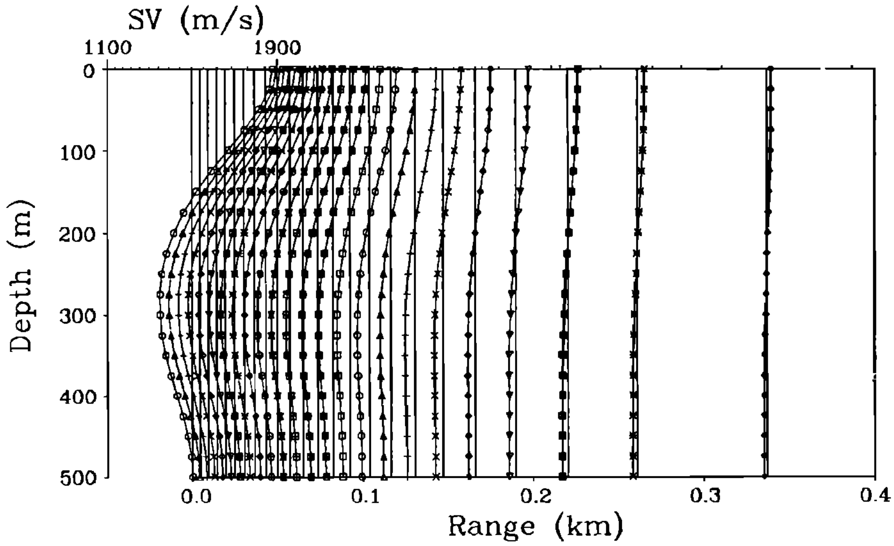

FIG. 2. Change in sound-speed profile with range for test problem 4. Note that the profile changes to isovelocity (1500 m/s) essentially within the first 400 m. The initial profile has a maximum speed of 1877 m/s at the surface and a minimum speed of 1346 m/s near middepth.

parabolic-approximation techniques (narrow- or wide-angle parabolic equations solved by split-step or finite-difference techniques); and finite-element/finite-difference solutions of the full wave equation.

The mode techniques provide approximate field solutions if implemented in the adiabatic approximation, while complete wave theory solutions can be obtained by including full mode coupling. We shall here apply a stepwise coupled mode code to obtain reference solutions for the various benchmark problems.

Parabolic approximations to the elliptic wave equation have been extensively studied over the past 10 years.[5] The advantage of using a parabolic wave equation is that it can be efficiently solved by noniterative forward marching techniques. However, any form of the parabolic equation is an approximate wave equation derived under the assumption of: (1) forward propagation only, and (2) that energy is propagating within a limited angular spectrum around the main propagation direction. There are two groups of PE codes, those using the split-step Fourier solution technique, and those using an implicit finite-difference scheme. Various forms of the parabolic equation can be solved with each technique (differences being associated with the angular spectral limitation), but here we shall exercise just the two most commonly used wide-angle PE codes.

The last category of models based on finite-difference and finite-element solutions of the full wave equation is well suited for providing benchmark solutions for propagation in general range-dependent environments. The existing codes, however, are extremely computer intensive and shall not be applied in this study.

A. Coupled modes (COUPLE)

A complete two-way solution for wave propagation in range-dependent fluid media can be formulated in terms of stepwise-coupled normal modes.[6] This technique consists in subdividing the environment into a number of range segments each with range-invariant properties, but with allowance for arbitrary variation of sound speed, density, and attenuation with depth. Hence, a continuously varying sound-speed profile with range would be approximated by a number of constant-profile segments, with small changes in profile from segment to segment. Similarly, a sloping bottom environment would be approximated by a series of constant depth segments, with slight changes in water depth from segment to segment. In both cases, the discretely segmented environment approaches the continuously varying environment for increasing number of range segments.

After having discretized the environment, the normal modes associated with each range segment are computed. By requiring continuity of pressure and horizontal particle velocity across segment boundaries and by imposing a known source condition at range zero together with a radiation condition at range infinity, a solution for the acoustic field based on propagator matrices can be constructed. The full solution is energy conserving and includes both forward and backscattered energy.

Most of the computational effort is associated with computing a full modal spectrum for each range segment. The approach selected in Ref. 6 is a discretization of the mode spectrum by introducing a pressure-release false bottom far below the water/bottom interface. This means that the full mode spectrum consists of proper guided modes (discrete spectrum), as well as of nonproper modes representing the continuous mode spectrum. The number of nonproper modes increases with false bottom depth, and only in the limit of an infinitely deep false bottom do we have an exact representation of the continuous mode spectrum. The modal eigenvalue problem is solved using the Galerkin method,[7] which transforms the search for complex eigenvalues into a tractable matrix eigenvalue problem by expanding the solution in terms of a basis set of isovelocity modes. This approach, however, requires the computation of a sufficiently large basis set to accurately represent the modes of the problem in each range segment. More details on the mathematical formulation and numerical implementation of the coupled-mode solution can be found in Refs. 6 and 7.

The particular numerical code COUPLE used in this study was developed in its original form by Evans.[8] However, several nontrivial modifications were introduced by us in order to be able to do all test problems. Thus the code was generalized to handle a multiprofile environment, to propagate a subset of the modes present at the source, to apply either a radiating or reflecting boundary condition at the source, and to provide results in either plane or cylindrical geometry.

The generation of accurate numerical solutions with COUPLE is a nontrivial exercise. Particular attention must be paid to a proper testing of the solution convergence with a number of user-specified input parameters. The most important ones are:

(1) Number of range segments used for representing a continuously varying profile or a constant bottom slope. This number must be increased until the solution stabilizes. As shown in Table III, we needed between 200 and 1000 range segments to solve the various benchmark problems accurately.

(2) Depth of the false bottom. This depth must be increased until the continuous mode spectrum is well sampled.

(3) Number of isovelocity basis modes used in the calculation. This number must be high enough to accurately represent the real modes of the problem.

With considerable care and attention, we obtained convergent solutions for all benchmark problems to a precision of the line thickness used in the graphical display of the results. Considering further that the coupled-mode solution intrinsically is a full-spectrum two-way solution of the wave equation, we have confidence that the COUPLE results presented in this paper are accurate numerical solutions of the posed mathematical problems.

B. Finite-difference PE (IFDPE)

The particular numerical code (IFDPE) used in this study was developed by Lee *et al.*[9-11] It solves a wide-angle parabolic equation due to Claerbout by implicit finite differences. The solution is set up for a finite depth domain limited above by a flat sea surface and below by a reflecting false bottom, which should be placed far below the water/bottom interface. To avoid reflections from the false bottom, a high bottom attenuation is introduced in the lower portion of the depth domain. The environment is discretized in depth and range, and starting from a known source-field distribution over depth at range zero, the solution can be advanced stepwise in range taking into account environmental changes as the solution progresses. Most of the computational effort is associated with advancing the solution in range, while environmental updates require little computational effort. Hence, it is possible to change sound-speed profile and water depth at each range step without a significant increase in calculation time.

The generation of accurate IFDPE results requires attention to several user-specified input parameters. The most important ones are:

(1) Starting field at source range. Since we generally attempt to model the field radiated by an omnidirectional point source, it is important to use a wide-angle source with a beamwidth that is compatible with the angular limitations inherent in the parabolic equation being solved. In the benchmark problems, we used Greene's wide-angle source[12] for test cases 2 and 3, and a modal source for case 4.

(2) False bottom depth. The false bottom must be placed deep enough that the field solution in the waveguide is not contaminated by spurious boundary reflections.

(3) Reference sound speed. In general, a reasonable choice is to set the reference speed equal to the average speed of the propagation duct. In multipath situations, a change in reference speed will cause slight shifts in the interference pattern, and a careful selection of this parameter can significantly improve the overall accuracy of the solution, particularly in wide-angle problems.

(4) Spatial discretization in range and depth. There is no simple rule for *a priori* determination of range and depth increments. Essentially, these parameters must be determined through a convergence test, where Δr and Δz are decreased until the solution stabilizes. Due to the fact that energy is propagating within a limited angular spectrum around the horizontal (principal propagation direction), we can use considerably larger range increments than depth in-

TABLE III. Numerical parameters and CPU times for the various benchmark solutions.

Test case	Model	Source field	Δr(m)	Δz(m)	z_{max} (m)	c_0(m/s)	# stair steps or profiles	# modes in COUPLE	CPU time on FPS-164
1	COUPLE	modal	⋯	⋯	1000	⋯	500	10	10 min
2	COUPLE	modal	⋯	⋯	3000	⋯	200	90	3×8 h
	IFDPE	Greene	5.0	1.0	4000	1500	⋯	⋯	7 min
	PAREQ	Greene	5.0	0.65	1333	1500	⋯	⋯	1 min
3	COUPLE	modal	⋯	⋯	3000	⋯	200	90	8 h
	IFDPE	Greene	5.0	0.5	2000	1500	⋯	⋯	7 min
	PAREQ	Greene	5.0	0.65	1333	1500	⋯	⋯	1 min
4A	COUPLE	modal	⋯	⋯	1000	⋯	1000	17	2.5 h
	IFDPE	modal	1.0	0.5	500	1700	1000	⋯	8 min
4B	COUPLE	modal	⋯	⋯	1000	⋯	1000	30	6.5 h
	IFDPE	modal	0.5	0.25	500	1700	1000	⋯	15 min

crements. As shown in Table III, we obtained stable solutions for the various test problems with $\Delta r = 2\text{–}10\ \Delta z$. However, the the spatial grid size is in all cases much smaller than an acoustic wavelength.

The IFDPE code was successfully applied to test problems 2, 3, and 4. It appears that this wide-angle code accurately handles propagation within $\pm 40°$ from the horizontal in the forward-scattered approximation. Moreover, the IFDPE is computationally efficient being 10–100 times faster than the coupled-mode code.

C. Split-step PE (PAREQ)

The classical parabolic solution technique is the split-step Fourier algorithm,[13,14] which is computationally efficient but can be applied only to a certain class of parabolic equations. Thus the wide-angle Claerbout equation is not solvable with the split-step technique. However, Thomson and Chapman[14] derived a moderately wide-angled equation, which is the basis for the split-step PE results shown in this study.

The actual computer code used (PAREQ)[15] is based on an environmental description with discretization in depth and range similar to that outlined in Sec. II B. The main difference is that the field solution here is advanced by doing successive Fourier transforms over depth. Again, the source-field distribution must be specified (Gaussian, Greene, or modal source) together with the false bottom depth and the reference sound speed. Finally, range and depth increments are determined through a convergence test. Essentially, the detailed comments made in Sec. II B for ensuring accurate solutions with the finite-difference technique, also apply to the split-step technique.

The PAREQ code was only used for solving test problems 2 and 3. The Thomson-Chapman equation is clearly more narrow angle than the Claerbout equation and consequently provides less accurate results for the relatively wide-angle test problems. It appears that the effective beamwidth is around $\pm 20°$ compared to $\pm 40°$ for the Claerbout equation. However, the PAREQ code is computationally efficient. Numerically convergent solutions of the test problems were obtained five–ten times faster than with the IFDPE code.

III. CASE 1: WEDGE WITH PRESSURE-RELEASE BOUNDARIES

Numerical parameters and CPU times for the various benchmark solutions are summarized in Table III. It is evident that the computational effort needed in each case depends on the required accuracy of the numerical solution. The results presented here are all accurate (numerically convergent) to within the line thickness used in the graphical displays of the results. The CPU times given in the last column of Table III refer to an FPS-164 attached processor. The corresponding times for two commonly used VAX computers (8600 and 780) are obtained by multiplying by a factor 4 for a VAX-8600 and by a factor 12 for a VAX-780.

Test case 1 is a wedge-shaped waveguide with pressure-release perfectly reflecting boundaries. Since sound propagating towards the apex will be completely backscattered, any attempt to solve this problem should be based on a two-way solution of the wave equation. Consequently, the parabolic equation codes were not considered, and results are provided only by the coupled-mode model.

A. COUPLE results

Propagation in the pressure-release wedge is characterized by the presence of 6 discrete modes at the source range. However, due to symmetry in the problem (source at mid-depth) only the odd-numbered modes are excited. Hence, the acoustic field is initially composed of 3 modes (1, 3, and 5) which propagate upslope towards the apex until first mode 5 reaches its critical depth and is turned around followed by modes 3 and 1, which reach their critical depths further upslope. Evidence of this mode stripping effect can be seen in Fig. 3(a), which displays the outgoing coupled-mode solution. In the first kilometer, we see a characteristic 3-mode interference pattern, followed by a 2-mode pattern up to around 2.2 km from the source, and then a single-mode region extending to a range of 3.4 km. No energy propagates beyond the critical depth for mode 1.

The full two-way solution is given in Fig. 3(b). This result is structurally similar to the one-way result, but with a higher mean level of 4–5 dB. We also notice the rapidly varying multipath structure due to interference between incoming and outgoing waves. For easier comparison with alternative solutions, we present in Fig. 4 an expanded view of the COUPLE two-way solution covering just the first kilometer.

As shown in Table III, a stable COUPLE solution be-

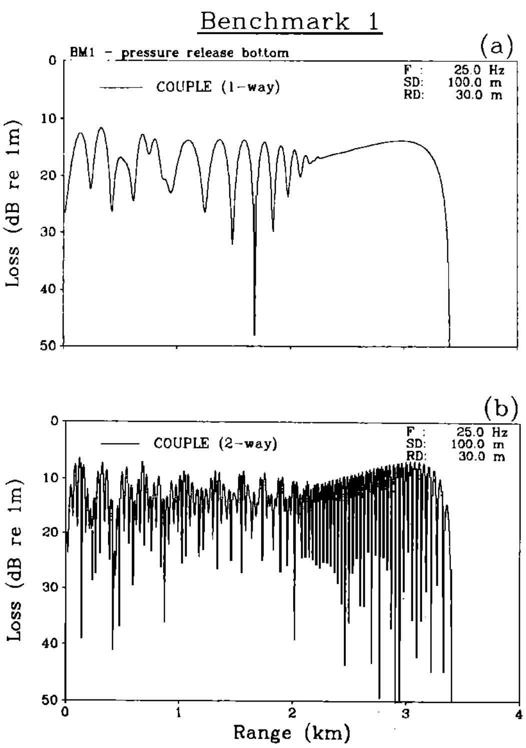

FIG. 3. Coupled-mode solutions for wedge with pressure-release bottom. (a) Outgoing solution only, (b) full two-way solution.

Benchmark 1

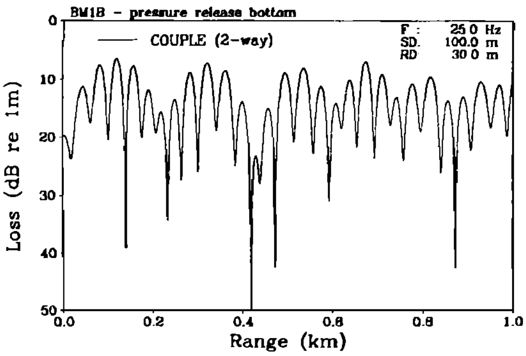

FIG. 4. Expanded view (0–1 km) of COUPLE two-way solution for wedge with pressure-release bottom.

yond the nearfield (> 100 m) was obtained by subdividing the sloping bottom into 500 stair steps and by including 10 modes in the computations (6 propagating modes and 4 non-propagating horizontally evanescent modes). To obtain convergence also in the nearfield, more evanescent modes need to be included, which, however, was not possible due to numerical overflow problems. Particular attention was paid to the correct handling of the pressure-release bottom boundary condition. By comparison with known reference solutions for range-independent environments, it was found that a pressure-release bottom was correctly simulated in COUPLE by using a high bottom speed of 10^{10} m/s and a low density of 10^{-4} g/cm^3, and with the false bottom placed at 1000-m depth. Full two-way solutions were obtained within 10 min on the FPS-164.

To assess the absolute accuracy of the COUPLE results presented in Figs. 3 and 4, we can compare our solutions to the reference provided by Buckingham and Tolstoy,[16] who formulated a solution for the pressure-release wedge in terms of the natural modes of the wedge. This is a conceptually simpler approach than used by COUPLE (no range discretization is required), which should provide accurate and reliable results for this particular propagation problem. The COUPLE results are found to be in excellent agreement with the reference solutions beyond the nearfield. Buckingham and Tolstoy[16] show that it is necessary to include around 100 evanescent modes to obtain accurate nearfield results. Based on the above comparison we can conclude that, except for very near the source (< 100 m), the COUPLE results in Figs. 3 and 4 are accurate to within a few tenths of a decibel.

IV. CASE 2: WEDGE WITH PENETRABLE LOSSLESS BOTTOM

Test case 2 is a wedge-shaped homogeneous ocean (1500 m/s) overlying a homogeneous fast bottom with a speed of 1700 m/s, a density of 1.5 g/cm^3, and an attenuation of 0.0 dB/λ. These bottom properties result in perfect reflection for waterborne-energy incident at angles up to the critical grazing angle of 28° (discrete mode spectrum), while energy incident at steeper angles (continuous mode spectrum) is subject to increasing reflection loss with angle, with a maximum loss of approximately 12 dB per bounce at normal incidence.

Since upslope propagation is characterized by the steepening of ray paths at each bottom bounce, all of the energy radiated by the source will eventually interact with the bottom at angles greater than the critical angle and penetrate into the bottom. Hence, upslope propagation in a wedge with a penetrable bottom is characterized by the forward radiation of sound into the bottom at the mode cutoff range, rather than by the backscattering of sound within the water column as was the case for the pressure-release wedge. Consequently, we are here dealing with a propagation situation that seems well suited for being modeled by parabolic equation codes. Coupled-mode solutions, however, are needed as a reference to check the importance of backscattering as well as of the angular spectrum limitations associated with the parabolic wave equations.

A. COUPLE results

One- and two-way COUPLE solutions for test problem 2 are shown in Fig. 5. For both receiver depths, the difference between one- and two-way results is seen to be approximately 2 dB at longer ranges. It should be pointed out that this level difference *does not* imply that backscattering is significant in this test problem. Instead, it shows that an approximate formulation (outgoing waves only) of the complete two-way solution can lead to significant errors in the computed sound levels (2 dB in this case). More precisely, we can state that *a one-way solution will generally not be an*

Benchmark 2

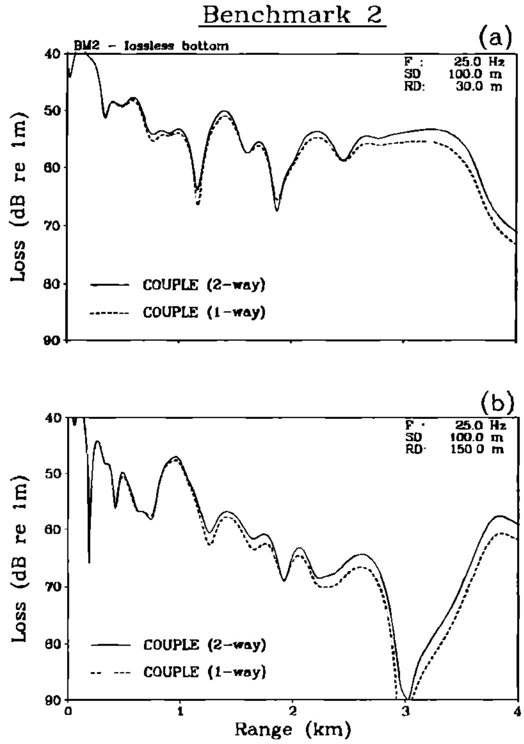

FIG. 5. Coupled-mode solutions for wedge with lossless penetrable bottom. (a) Receiver at 30 m, (b) receiver at 150 m. Note the level difference between one-way and two-way results of about 2 dB at longer ranges.

accurate representation of the outgoing-wave component of the full two-way solution. Evidence of the lack of back-scattered energy for the penetrable-wedge problems was first shown in the ray solutions generated by Westwood[17] and subsequently confirmed by us by displaying separately the outgoing and incoming wave components of the full two-way COUPLE solution. The backscattered field is indeed very weak in this case being around 40 dB lower in level than the forward-propagating field component. In this context, it is interesting to note that the IFDPE formulation handles one-way propagation in the same approximation as the one-way coupled-mode solution (Fig. 6).

Propagation in test problem 2 is characterized by the presence of three discrete modes at the source range. However, to produce a full-spectrum solution, it was necessary to compute a total of 90 modes, 3 proper discrete modes, and 87 nonproper modes, with the false bottom placed at a depth of 3000 m. The primary difficulty encountered in obtaining stable numerical results for this test problem was associated with the eliminations of reflections off the false bottom. Since there is no loss in the real bottom, energy radiated at steep angles from the source will travel down to the false bottom and back to the water without attenuation and hence contaminate the solution.

We found that the only practical solution to this problem was to limit the beamwidth of the source. Thus, if we consider a maximum beamwidth of $\pm 35°$, which means limiting the number of computed source modes, the steepest traveling energy when reflected off the false bottom will appear in the water column beyond a range of 4 km and therefore not contaminate our results. However, such a solution is inaccurate at short ranges from the source where high-angle energy ($> 35°$) contributes significantly to the field solution. It turned out that for a false bottom at depth 3000 m and with a $\pm 90°$-source beamwidth, spurious reflections appear at a range of approximately 260 m from the source. If we now limit the beamwidth to $\pm 60°$ the false-bottom reflections are shifted out in range to around 1200 m from the source, while a $\pm 35°$ beamwidth does not produce false reflections within our observation range. By carefully piecing together the full solution from the above three range-limited solutions (with overlap points chosen where adjacent solutions agreed to within 0.1 dB), we obtained the results displayed in Fig. 5. As shown in Table III, this benchmark solution required 8 h of CPU time on the FPS-164 for each source beamwidth, using a subdivision of the slope into 200 stair steps.

Even though the full-spectrum COUPLE solution for this test problem was construction from three partial solutions, we are confident that the two-way results given in Fig. 5 are accurate to within a fraction of a decibel. This conclusion is based on the fact that the penetrable wedge represents a slightly more benign environment for the solution technique employed in COUPLE than the pressure-release wedge, for which we obtained accurate results to within a few tenths of a decibel.

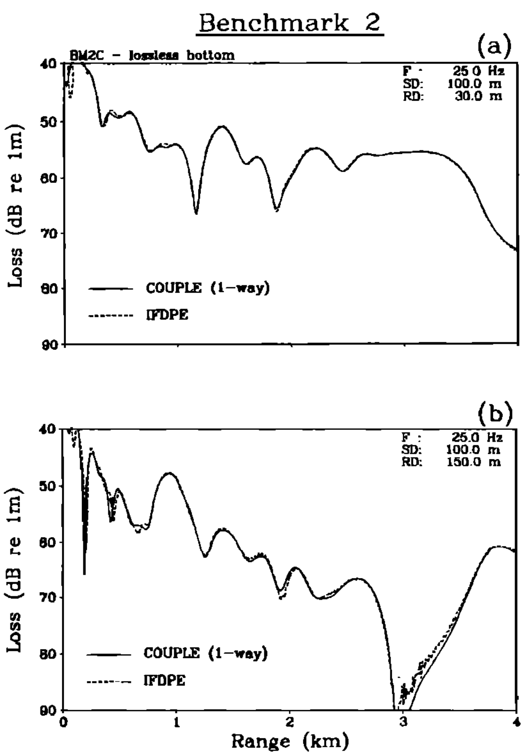

FIG. 6. Comparison of one-way results from COUPLE and IFDPE for wedge with lossless penetrable bottom. (a) Receiver at 30 m, (b) receiver at 150 m.

B. IFDPE results

IFDPE solutions for the penetrable wedge are compared with one-way COUPLE results in Fig. 6. The agreement is excellent indicating that one-way propagation in this environment is handled well by the wide-angle Claerbout equation. Numerically convergent results were obtained with the parameters given in Table III in just 7 min on the FPS-164. Note that the false bottom was placed at a depth of 4000 m in an attempt to avoid spurious reflections. We succeeded except in the deep interference null at range 3 km in Fig. 6(b), where the IFDPE solution is clearly noisy. This problem could have been solved by moving the false bottom still further down, but array limitations in the code precluded this solution. The primary reason for having less problems with false bottom reflections in the IFDPE solution than in the coupled-mode solution, is the presence of a deep artificial attenuation layer in the PE environment introduced to simulate a lower-boundary radiation condition (see Sec. II B).

C. PAREQ results

More narrow-angle parabolic equation results obtained from PAREQ are compared with one-way COUPLE solutions in Fig. 7. The agreement here is less satisfactory, indicating that the angular limitations associated with the Thomson–Chapman equation ($\pm 20°$) are too restrictive for this test problem. This conclusion might have been anticipated considering that the radiation of sound into the bottom during mode cutoff is associated with propagation angles above the critical grazing angle of 28°. Hence, an ac-

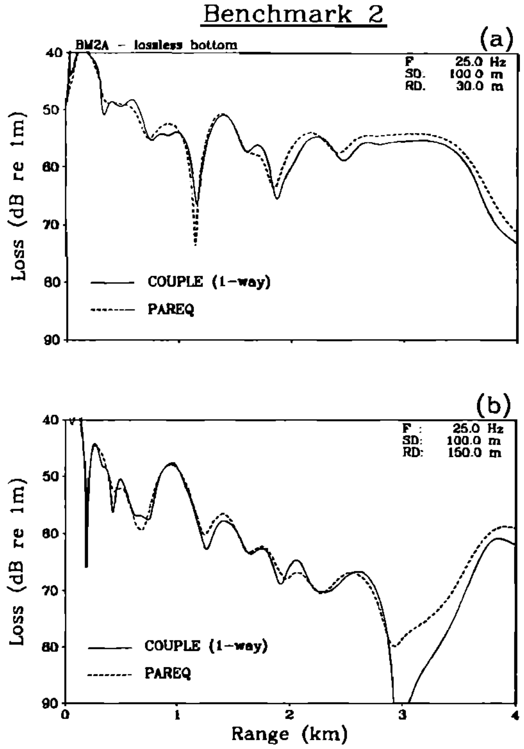

FIG. 7. Comparison of one-way results from COUPLE and PAREQ for wedge with lossless penetrable bottom. (a) Receiver at 30 m, (b) receiver at 150 m.

curate solution of this test problem requires a wide-angle PE capability similar to that offered by the Claerbout equation.

V. CASE 3: WEDGE WITH PENETRABLE LOSSY BOTTOM

This is a more realistic ocean acoustic problem where a bottom attenuation of 0.5 dB/λ has been included. Otherwise, parameters are the same as in case 2.

A. COUPLE results

An illustrative 2-D field solution generated by the coupled-mode code is shown in Fig. 8. We notice that radiation into the bottom is particularly evident at short ranges and around 3.5 km where the fundamental mode is cut off. Also shown are horizontal dashed lines corresponding to standard transmission loss displays for receivers at 30- and 150-m depth.

One- and two-way COUPLE solutions for test problem 3 are shown in Fig. 9. These results are very similar to the ones obtained for the lossless penetrable bottom in case 2. Again, we see that the one-way approximation results in 2-dB errors in computed levels at longer ranges.

Due to the presence of a realistic bottom loss of 0.5 dB/λ, steep-angle energy reflected off the false bottom at 3000-m depth will undergo sufficient attenuation (-40 dB) so as not to contaminate the results in the upper 150 m of the solution domain. Consequently, we can do a full-spectrum ($\pm 90°$) COUPLE solution in a single solution pass.

Stable numerical results to within the line thickness used in the graphical displays were obtained by including 90 modes and by subdividing the slope into 200 stair steps. As shown in Table III, this calculation required 8 h of CPU time on the FPS-164. Since the numerical solution of this case presented fewer complications than for test problem 2, we are confident that the two-way results presented in Fig. 9 constitute a reference solution to test problem 3 to within an accuracy of a few tenths of a decibel.

B. IFDPE results

IFDPE solutions for test problem 3 are compared with one-way COUPLE results in Fig. 10. The agreement is excellent indicating that one-way propagation in this environment is accurately handled by the wide-angle Claerbout equation. Numerically convergent results were obtained with the parameters given in Table III. Note the much lower computational effort required to generate a parabolic equa-

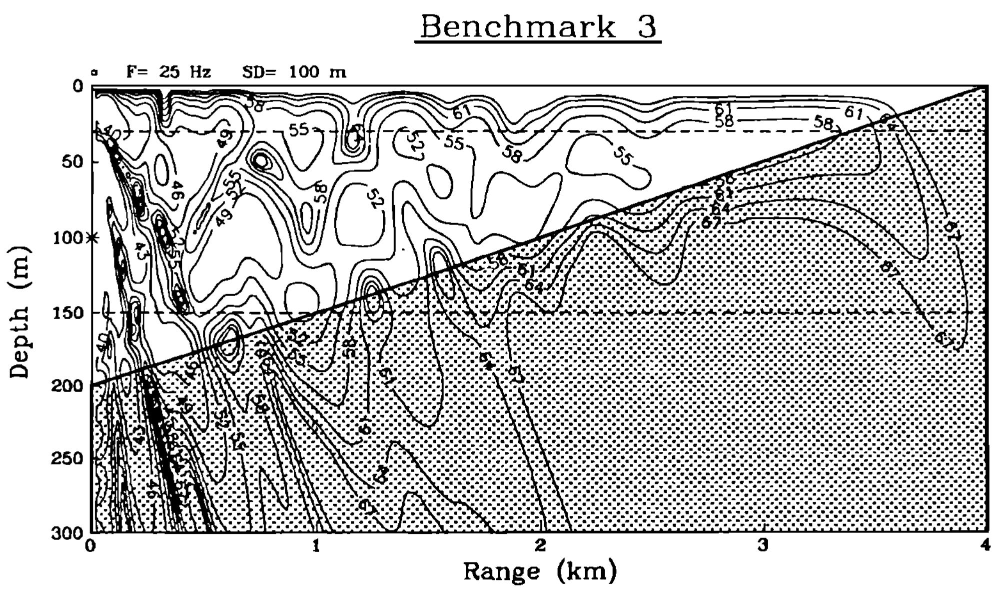

FIG. 8. Contoured field solution (COUPLE) for wedge with lossy penetrable bottom.

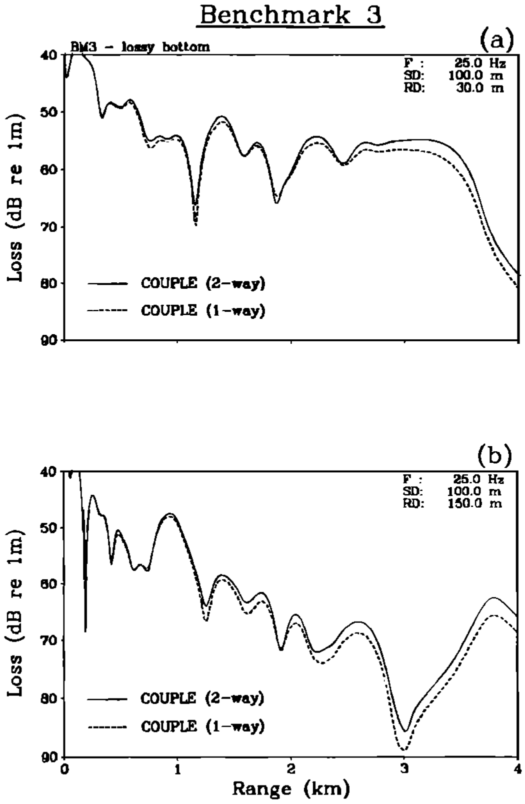

FIG. 9. Coupled-mode solutions for wedge with lossy penetrable bottom. (a) Receiver at 30 m, (b) receiver at 150 m. Note the level difference between one-way and two-way results of about 2 dB at longer ranges.

tion result compared to a full-spectrum two-way coupled-mode result.

C. PAREQ results

A comparison of narrow-angle PAREQ results with one-way COUPLE reference solutions are given in Fig. 11. Here the agreement is less satisfactory indicating that the angular-spectrum limitations associated with the Thomson–Champman equation ($\pm 20°$) are too restrictive for this test problem. The PAREQ results were generated in less than 1 min on the FPS-164 using the numerical parameters given in Table III.

VI. CASE 4: PLANE-PARALLEL WAVEGUIDE

We are here considering an inhomogeneous waveguide limited above by a pressure-release surface and below by a rigid bottom. As shown in Fig. 2, the range dependence is entirely associated with the varying sound-speed profile with range. We shall present COUPLE and IFDPE results for two different source beamwidths, a beam-limited source of width $\pm 43°$ obtained by summing the first 10 source modes, and an omnidirectional source obtained by summing all 17 source modes.

A. COUPLE results

One- and two-way COUPLE solutions for test problem 4A (10 source modes) are shown in Fig. 12(a), while results for case 4B (17 source modes) are given in Fig. 13(a). The small differences between one- and two-way solutions in Fig.

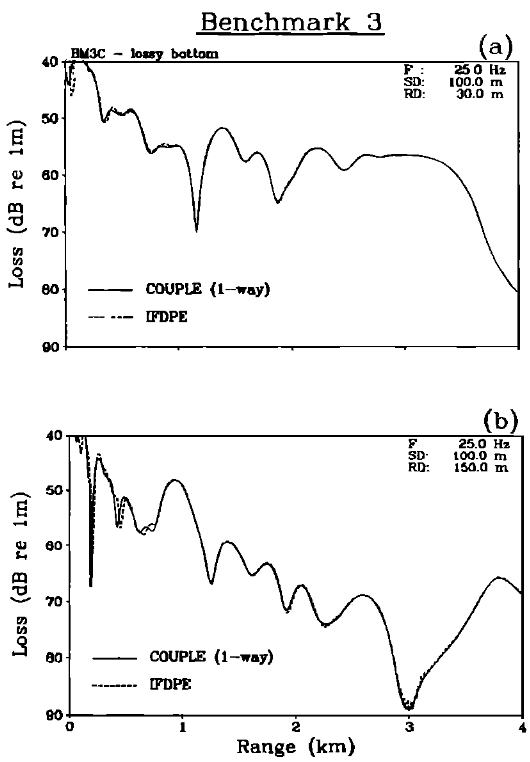

FIG. 10. Comparison of one-way results from COUPLE and IPDPE for wedge with lossy penetrable bottom. (a) Receiver at 30 m, (b) receiver at 150 m.

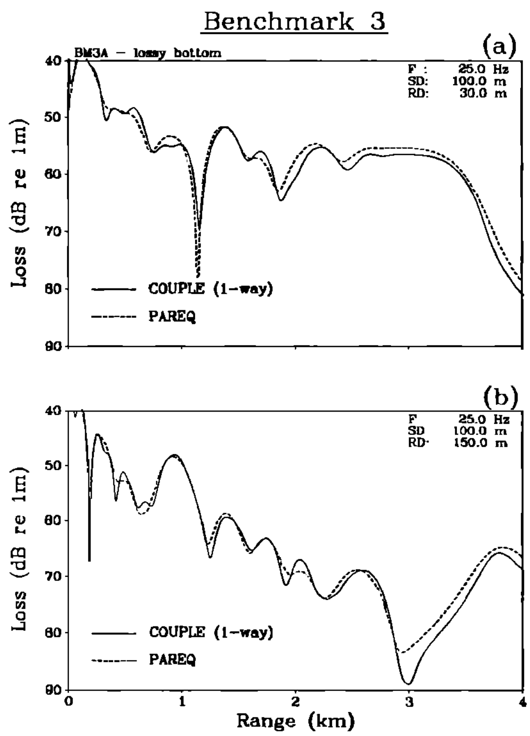

FIG. 11. Comparison of one-way results from COUPLE and PAREQ for wedge with lossy penetrable bottom. (a) Receiver at 30 m, (b) receiver at 150 m.

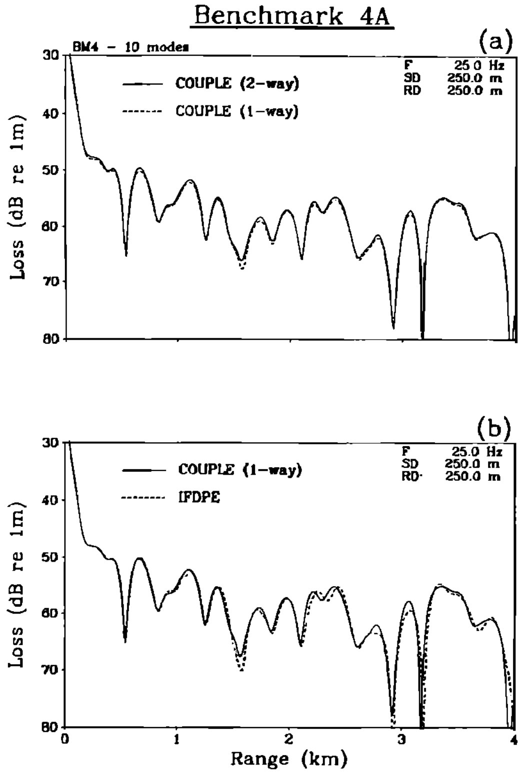

FIG. 12. Solutions for plane-parallel waveguide with rigid bottom using 10 source modes only. (a) Coupled-mode solutions, (b) comparison of one-way results from COUPLE and IFDPE.

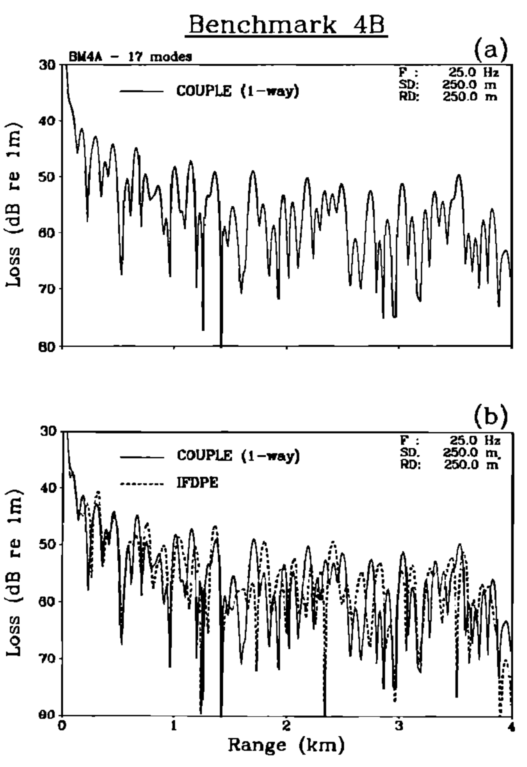

FIG. 13. Solutions for plane-parallel waveguide with rigid bottom using all 17 source modes. (a) Coupled-mode solution, (b) comparison of one-way results from COUPLE and IFDPE.

12(a) are probably within the overall numerical accuracy of the solutions.

As shown in Table III, stable COUPLE solutions were obtained by approximating the continuous sound-speed variation in range with 1000 discretely changing profiles. Moreover, since profile changes are strongest near the source, we introduced a nonequidistant range segmentation. To obtain stable numerical results for the 10-mode problem, we had to include 17 isovelocity basis modes in the calculations. Likewise, 30 basis modes were needed to solve the 17-mode problem. Particular attention was paid to the correct handling of the rigid bottom boundary condition. By comparison with known reference solutions for range-independent environments, it was found that a rigid bottom was correctly simulated in COUPLE by using a high speed (10^{10} m/s) as well as a high density (10^5 g/cm^3), and with the false bottom placed at 1000-m depth. Full COUPLE solutions required 2.5 h of CPU time for case 4A and 6.5 h for case 4B.

To assess the absolute accuracy of the COUPLE results presented in Fig. 12(a), we can compare our solutions to the reference provided by Thomson *et al.*,[18] who numerically implemented a solution derived by DeSanto[19] for this particular propagation problem based on conformal mapping techniques. This reference solution essentially treats a range-independent homogeneous-waveguide problem in mapped coordinates and, hence, circumvents the requirement for a discretization of the sound-speed profile in range. The COUPLE results are found to be in excellent agreement with the reference solutions,[18] again confirming the reliability and correctness of the coupled-mode results. Based on the above comparison, we can conclude that the COUPLE benchmark solutions presented in Figs. 12(a) and 13(a) are accurate to within 1 dB.

B. IFDPE results

The IFDPE solution for the beam-limited source (10 modes) is compared with the one-way COUPLE result in Fig. 12(b). The agreement is very good indicating that this propagation situation is well handled by the wide-angle Claerbout equation. Best agreement was achieved with a PE-reference sound speed of 1700 m/s.

Results for the omnidirectional source (17 modes) are compared in Fig. 13(b). Here, there is poor agreement in the multipath-interference structure due to the angular limitations inherent in the parabolic wave equation, and no improvement was possible by changing the reference sound speed.

These two test problems essentially confirm the angular spectral limitation of $\pm 40°$ in the Claerbout equation. Numerically convergent results were obtained with the parameters given in Table III. Note that wide-angle propagation situations generally require smaller spatial solution grids for numerical accuracy. The IFDPE code is again seen to be computationally efficient (8 and 15 min for the two test problems) compared to the coupled-mode code.

VII. SUMMARY AND CONCLUSIONS

We have presented solutions for four different range-dependent ocean acoustic problems chosen as benchmarks for validating general-purpose numerical codes. Apart from the important observation that *one-way* wave equations do not provide accurate results for propagation over sloping bottoms, the performance of the three different acoustic models exercised on the test problems can be summarized as follows.

(1) *COUPLE* provides a full-spectrum two-way solution of the elliptic-wave equation based on stepwise-coupled normal modes. This code is ideally suited for providing benchmark results in general range-dependent ocean environments. The solution technique is computationally slow and therefore impractical at higher frequencies, but results for the four benchmark problems were generated with an estimated accuracy of less than 1 dB.

(2) *IFDPE* provides a limited-spectrum one-way solution of the parabolic approximation to the full wave equation. The implicit finite-difference solution technique is computationally efficient, and accurate one-way results are provided for energy propagating within $\pm 40°$ with respect to the horizontal.

(3) *PAREQ* provides a narrow-angle one-way solution of the parabolic approximation to the full wave equation. The split-step Fourier solution technique is computationally efficient, and accurate one-way results are provided for energy propagating within $\pm 20°$ with respect to the horizontal.

In conclusion, we feel that the benchmark solutions presented here should be useful in future model validation tests. Moreover, the reported calculation times, though not optimized in this study, could be considered a point of reference for future model developers. Thus any code that can reproduce the results published here with significantly less computational effort truly represents a progress in range-dependent ocean acoustic modeling.

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